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Zak A. Combinatorial Actions In Elementary School

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Abstract. *The article presents an experimental study aimed at studying the possibilities of improving combinatorial actions in elementary school, in particular among second-grade students. 110 students took part in the study: the control group consisted of 53 people, the experimental group - 57 people with whom 32 additional extracurricular activities were conducted under the non-curricular content program "Combinatorics". As a result of the experiments, it was shown that additional classes are an essential condition for improving combinatorial actions in second-graders.*

Keywords: *combinatorial actions, the Combinatorics program, additional extracurricular activities, second grade students, search tasks of three types: comparative, spatial, route.*

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1. Introduction.

One of the directions in preparing younger schoolchildren for studying in the middle grades of school is to improve combinatorial actions. Our work shows that successful learning of mathematics presupposes that children have developed actions of a combinatorial nature [8, p.261 – 285].

This experimental work is aimed at determining the conditions that contribute to the formation of combinatorial actions in schoolchildren studying in the second grade of primary school.

1.1.Study of combinatorial actions in primary school students.

English L.D. studied the features of finding solutions in combinatorial problems of varying complexity. A 1991 study compared the characteristics of actions performed by 7-year-old children and 4-6-year-old children. The results of the experiments indicate that 7-year-old children more often use a systematic approach to finding a solution associated with variation than 5-year-old children.

In a 1993 study , combinatorial problems were solved by schoolchildren aged seven to twelve. It turned out that, as in the 1991 experiments, 7-year-old children actually use a systematic strategy within which 3 parameters vary.

A 2005 study summarizes previously conducted experiments: it characterizes the typology of types of combinatorial problems and discusses the question of how children's abilities correlate with how difficult they are asked to solve combinatorial problems.

Krpec, R. (2014) studied how children act when ordering, especially the characteristics of actions associated with ordering objects of various kinds, in particular, letters and pictures. It was found that as early as 7 years old, children successfully organize marked objects using one strategy or another.

Features of solving problems of combinatorial type, classified as simple, were studied by Maher, C., & Yankelewitz, D. (2010). The study identified the methods by which children could justify the solutions they found, as well as the methods used by children to organize the content of the proposed problems.

The work of White H. (1984) studied how children aged 8–11 years cope with the well-known tasks of J. Piaget associated with combining solutions. The study made it possible to establish the cognitive capabilities of children, which are realized when the required result is achieved.

Poddiakov A.N. (2011) studied how 7-year-old and 4-6-year-old children experiment in combination conditions. The possibilities of improving combinatorial actions when playing with objects that are characterized as multidimensional were established.

1.2. Characteristics of the work.

Consideration of the above-mentioned works, which studied the characteristics of solving combinatorial problems by primary schoolchildren, shows that experiments are carried out, as a rule, on problems associated with learning at school.

We believe that children's actions in solving combinatorial problems can also be studied using the material of non-educational problems.

This approach will make it possible to include a larger number of children in the contingent, in particular, those who have low grades in academic subjects. However, they can successfully solve non-academic tasks where academic knowledge is not critical.

Our work was aimed at identifying conditions that will contribute to the improvement of combination actions among schoolchildren studying in the second grade. Our work was aimed at identifying conditions that will contribute to the improvement of combination actions among schoolchildren studying in the second grade. At the same time, we relied on the assumption that conducting additional extracurricular classes in the Combinatorics program creates the specified conditions.

The Combinatorics program includes three types of combinatorial problems based on non-educational material. To achieve success in solving type 1 problems, it is necessary to use a combination of features of the proposed objects (these are comparative problems). In problems of the 2nd type, combination is implemented to transform the proposed order of objects into the required one (these are spatial problems). In problems of the 3rd kind, a combination of movements of some conditional character is performed (these are route problems).

Our work included 3 stages.

At stage 1, a control (53 people) and an experimental (57 people) groups were formed.

To diagnose combination actions, subjects in the control and experimental groups solved combinatorial problems in which possible movements were combined along the lines of the playing field from one point to another.

At stage 2, 32 extracurricular activities were conducted with subjects from the experimental group. In these classes, held weekly, schoolchildren solved problems included in the Combinatorics program material.

At stage 3, after all extracurricular activities, schoolchildren of both groups solved the same problems that were offered to them at stage 1, before extracurricular activities.

2. Materials and methods.

Based on the Combinatorics program, 32 additional extracurricular classes were conducted. In these classes, children solved problems of a comparative nature (9 independent types), problems of a spatial nature (10 independent types) and problems of a route nature (13 independent types). It is important to note that in one lesson the children solved problems of only one type of the corresponding kind.

2.1. Views of comparative problems.

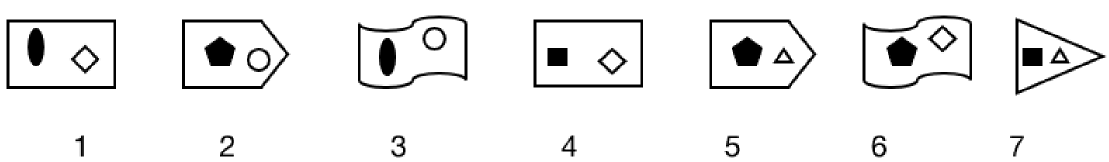

Fig. 1. Windows.

View 1, specifically: “Is window 2 or 3 shaped like window 6?”

View 2, in particular: “Does Window 1 or 3 have the same feature as Window 5?”

View 3, specifically: “Does Window 4 or 5 share fewer features with Window 1?”

View 4, in particular for example: “Window 2 or 3 has a shape like window 6, but a small dark figure like window 1?”

View 5, in particular: “Does Window 1 or 3 have the same feature as Window 6?”

View 6, in particular: “Windows 1 and 6 have the same feature. Which two windows, 2 and 3 or 1 and 4, have more of the same features than windows 1 and 6?

View 7, in particular: “Does window 3 or 5 have the same shape as window 1, a small dark figure like window 6, and a small light figure like window 2?”

View 8, in particular: “Does Window 4 or 3 have one feature in common with Window 1, one feature in common with Window 8, and one feature in common with Window 6?”

View 9, in particular: “Windows 2, 5, 6 have one identical feature. Which three windows have – 2, 3, 5; 1, 4, 6 or 5, 6, 7 - have as many identical features as windows 2, 5, 6?”

Each lesson included solving 3 variants of problems of the type that were proposed in this lesson: in 1 variant the answer was unknown, in 2 - the question to the proposed conditions was unknown, in 3 - based on one part of the conditions and the question, it was necessary to find another part of the conditions.

The first option is suggested in the nine examples presented above. The second option, in particular: “Windows 2, 3, 6. Choose a question for which the answer “At window 2” will be correct: a) Which window has a small dark figure the same as window 4?; b) Which window has a small light figure the same as window 3?; c) Which window has the same shape as window 5?”

The third option, in particular: “Windows 3 and 6. Which window has a small light figure like window 6?” Which third window will answer this question: a) window 2; b) window 5; c) window 4.

2.2. Views of spatial problems.

View 1, in particular: “How to arrange the numbers |4|__|8| is it possible to convert using two actions to obtain the arrangement |8|4|__|?”

View 2, in particular: “How can the order of letters $\boxed{\begin{array}{ll} р & к \\ м & \end{array}}$ be transformed using two actions to obtain the order $\boxed{\begin{array}{ll} & р \\ м & к \end{array}}$?”

View 3, in particular: “How is the order of the letters |S|__|R|__|T| is it possible to convert using two steps to obtain the order |_|S|R|T|__|?" The solution to problems of the first three views is carried out according to the rule: one action consists of rearranging any sign (number or letter) into an empty space.

View 4, in particular: “How can a PMK sign arrangement be converted using two actions into a KPM arrangement?”

View 5, in particular: “How can the arrangement of letters $\boxed{\begin{array}{ll} р & к \\ м & т \end{array}}$ be transformed using two actions into an arrangement $\boxed{\begin{array}{ll} т & р \\ м & к \end{array}}$?”

The solution to problems of the last two views is based on the rule: each action consists of simultaneous exchange of places of any two characters (letters or numbers).

View 6, in particular: “How to arrange the letters P| |S|R| it is possible to convert using two actions to obtain the arrangement of numbers |7|7|_|4| ?

View 7, in particular: “How can the arrangement of letters $\begin{array}{|cc|}\hline \text{р} & \text{m} \\ & \text{м} \\ \hline\end{array}$ be converted using 2 actions to obtain the order of numbers $\begin{array}{|cc|}\hline 5 & \\ 8 & 5 \\ \hline\end{array}$?”

View 8, in particular: “How can the order of the letters P P M K be converted using two operations to obtain the order of the digits 6 8 5 5?”

View 9, in particular: “How can the order of letters $\begin{array}{|cc|}\hline \text{р} & \text{к} \\ \text{р} & \text{т} \\ \hline\end{array}$ be converted using two actions to obtain the order of numbers $\begin{array}{|cc|}\hline 9 & 4 \\ 7 & 4 \\ \hline\end{array}$?”

View 10, in particular: “How is the arrangement of letters |C|__|C|__|T| it is possible to convert using 2 steps to obtain the arrangement of numbers |_|6|6|3|__| ?

Solving problems of the last five views (from view 6 to view 10) is based on the rule: after 2 actions, the same letters must be where the same numbers are.

Each lesson included solving 3 variants of problems of each of 10 views: in 1, the answer is unknown, in 2, the question to the proposed conditions is unknown, in 3, you need to find another part of the conditions based on the known part of the conditions of the problem and its question.

Option 2 is implemented in ten views proposed above. The second option, in particular: “What will be the arrangement: (a) C|__|P| or (b) |__|C|P|, if in the arrangement |P|__|C| perform 2 actions?

Option 3, in particular: “What was the arrangement at first: a)|__|C|P| or b)|C|__|P|, if after 2 actions the arrangement is |__|P|C| ?

2.2. Views of route problems.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Fig. 2. Playing field 1.

View 1, in particular: “What 2 steps did the chicken take to get from 11 to 18?”

The conditional character “Chicken” can move in a square across cells in different ways: a) she can take a straight step vertically, i.e. to an adjacent cell up or down (in particular: from 13 to 8 or 18) or horizontally, to an adjacent cell to the right or left (in particular, from 13 to 14 or 12); b) she can take an oblique step towards the adjacent number, i.e. from 13 to 7 or 9 or 19 or 17; 3) she cannot perform two identical steps in a row.

Solution: 11 – 12 – 18.

View 2, in particular: “What 2 jumps did the rabbit make to get from 11 to 5?”

The conditional character “Rabbit” can move in a square across cells in different ways: a) he can make a direct jump vertically (in particular: from 13 to 3 or to 23) or horizontally (in particular, from 13 to 11 or 15); b) he can do an oblique jump, in particular: from 13 to 5 or 1 or 21 or 25; c) he cannot perform 2 identical jumps in a row.

Solution: 11 – 13 – 5.

View 3, in particular: “What 2 jumps did the cat make to get from 11 to 19?”

The conditional character “Cat” can move in a square along the cells. The cat's movement is complex: it involves a straight jump through the cell and a straight step into the adjacent cell, in particular, from 13 to 6 or 2 or 4 or 10 or 20 or 24 or 22 or 16. The cat's movement is a jump with a turn.

Solution: 11 – 22 – 19.

View 4, in particular: “What two movements did the chicken and the rabbit make to get from 7 to 20?” In this problem, the chicken and the rabbit move one after the other, the chicken only walks straight, the rabbit only jumps obliquely. In particular: chicken: 12 - 7, rabbit: 7 - 19, chicken: 19 - 18, rabbit: 18 - 10.

Solution: 7 – 8 – 20.

View 5, in particular: “What two movements did the chicken and the rabbit make to get from 8 to 20?” In this problem, the chicken and the rabbit move one after the other, the chicken only steps diagonally, the rabbit only jumps straight. In particular: chicken: 8 - 14, rabbit: 14 - 4, chicken: 4 - 10, rabbit: 10 - 20.

Solution: 8 – 7 – 19.

View 6, in particular: “What two movements did the chicken and the cat make to get from 2 to 10?” In this problem, the chicken and the cat move one after the other, the chicken only walks straight, the cat jumps with a turn.

Solution: 2 – 3 – 10.

View 7, in particular: “What two movements did the chicken and the cat make to get from 4 to 18?” In this problem, a chicken and a cat move one after the other, 2) the chicken steps only obliquely, 3) the cat performs a turning jump.

Solution: 4 – 8 – 18.

View 8, in particular: “What two movements would the rabbit and the cat make to get from 18 to 5?” In this problem, the rabbit and the cat move one after the other, the rabbit only jumps straight, the cat makes a jump with a turn.

Solution: 18 – 8 – 5.

View 9, in particular: "What two movements did the rabbit and the cat make to get from 7 to 22?" In this problem, the rabbit and the cat move one after the other, the rabbit jumps only diagonally, the cat jumps with a turn.

Solution: 7 – 19 – 22.

View 10, in particular: "What four movements did the chicken and the rabbit make to get from 11 to 10?" In this problem, a chicken and a rabbit move one after the other, the chicken walks straight and diagonally, the rabbit jumps straight and diagonally.

Solution: 11 –12 – 24 – 20 – 10.

View 11, in particular: "What four movements did the chicken and the cat make to get from 6 to 4?" In this problem, a chicken and a cat move one after the other, the chicken takes straight and oblique steps, the cat makes jumps with a turn.

Solution: 6 – 12 – 19 – 15 – 4.

View 12, in particular: "What four movements did the rabbit and the cat make to get from 1 to 3?

In this problem, a rabbit and a cat move one after the other, the rabbit jumps straight and diagonally, the cat jumps with a turn.

Solution: 1 – 11 – 18 – 10 – 3.

View 13, in particular: "What four movements did the chicken, the rabbit and the cat make to get from 6 to 22?" In this problem, a chicken, a rabbit and a cat move one after another, the chicken walks straight and diagonally, the rabbit jumps straight and diagonally, the cat makes jumps with a turn.

Solution: 6 – 7 – 19 – 22.

Each lesson included solving 3 variants of problems: in 1, the answer was unknown, in 2, the question to the proposed conditions was unknown, in 3, based on one part of the conditions and the question, it was necessary to find another part of the conditions. Option 1 is proposed in the thirteen examples given.

Option 2, in particular: "Where can the hen get to 25 or 14 if she saddles two steps from 17?"

Option 3, in particular: "Where can the chicken get to 19 if it takes two steps from 25, from 21 or from 10?"8 2.

5.Characteristics of additional classes.

Each additional lesson in the Combinatorics program included 3 episodes. In episode 1, which lasted a quarter of an hour, the teacher and students discuss problem solving issues. This discussion allows students to figure out how to proceed to find a solution.

Schoolchildren become familiar with the steps of analyzing the conditions of a proposed problem and with the means by which management and control of the actions necessary to find a solution are carried out.

In episode 2, which lasted half an hour, schoolchildren perform independent work, during which they are offered 13–14 tasks. They can be successfully solved if you use the knowledge gained at the beginning of the lesson.

In episode 3, which lasted a quarter of an hour, the teacher and schoolchildren check solved problems, consider incorrect answers and again work out ways to analyze the conditions of the proposed problems and techniques that can be used to manage the solution of problems.

2.6. Diagnosis of combination actions.

Before conducting additional extracurricular activities and after their completion, a diagnostic session was organized to determine the characteristics of combination actions. Subjects from both groups solved combinatorial problems related to finding combinations of movements from one point of the playing field to another.

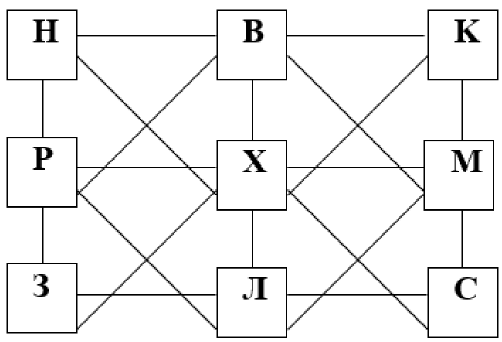

Fig. 3. Playing field 2.

The lesson began with the teacher talking about the playing field: “The squares represent houses with letters. A line is drawn from square to square. It marks a path along which you can move between letters.”

Then on the blackboard the teacher wrote the condition of a simple combinatorial problem: P --- ? --- B and addressed the children: “In this task you need to find two paths along which it is possible to go from P to B.”

Next, the teacher and children discussed possible solutions.

The discussion showed that two solutions are possible:

1) P --- X --- B 2) P --- N --- B

Then the schoolchildren received the conditions of two tasks, where they were required to discover possible combinations that included three movements (three links) between letters:

1. Q --- ? --- ? --- L. 2. M --- ? --- ? --- R.

No more than 10 minutes were allotted to solve each problem.

The characteristics of the proposed solutions were due to the fact that the subsequent combination, when compared with the previous one, can be chosen randomly and non-randomly. With a random selection, two adjacent combinations do not have a common link, for example: (B – H – R – L) or (B – X – M – L). With a non-random choice, adjacent combinations of movements contain a common link, for example: (B – N – R – L) and (B – N – M – L).

When any two neighboring combinations do not have a common link (random choice), this means that the solution to the proposed problem is implemented using a random strategy.

When any two neighboring combinations have a common link (non-random choice), this means that the solution to the proposed problem is implemented using a systematic strategy.

When some neighboring combinations do not have a common link, and other neighboring combinations have a common link, this means that the solution to the proposed problem is implemented using a mixed strategy.

As a result of processing and analyzing solutions to both problems, data was obtained that served as the basis for identifying three subgroups of children in the control and experimental groups.

The subjects who made up subgroup A used a random strategy when solving both problems. The subjects who made up subgroup B used a random strategy when solving the first problem, and a mixed strategy when solving the second problem. The subjects who made up subgroup B used a mixed strategy when solving both problems.

3. Results. The number of children who made up subgroups A, B and C in both groups of subjects during preliminary and final diagnostics is presented in the table

Table. Number of children in subgroups A, B and C (in %).

Groups	Diagnostic period					
	September			May		
	Subgroups of subjects					
	А	Б	В	А	Б	В
Control	48,2	30,4	21,4	32,1	28,6	39,3*
Experimental	50,0	31,0	19.0	15,5	24,2	60,3*

Note: * $p < 0.05$.

Consideration of information on the number of children in subgroups A, B and C of both groups of subjects provides grounds for characterizing changes in the results of their performance of diagnostic tasks that occurred from September (beginning of additional classes) to May (completion of additional classes).

So, according to the results of diagnostics before the start of classes, the differences between subgroups A, B and C of different groups of subjects (control and experimental) were insignificant, respectively: 54.7% - 52.6%, 32.1 - 36.8%, 13. 2% – 10.5%.

In May, the differences between the considered subgroups became more significant. In particular, the number of subjects in the control group of subgroups A and B exceeds the number of subjects in the experimental group of subgroups A and B–, respectively: 39.6% - 28.1%, 34.0% - 29.8%.

The greatest differences in May are characteristic of the subjects of the control and experimental groups, who made up subgroup B, respectively: 26.4% and 42.1% (the noted differences are statistically significant at $p < 0.05$). .

An interesting fact characterizing the changes that occurred in May in relation to September is that the number of subgroup A in both groups of subjects decreased: in the control group by 15.1%, and in the experimental group even more - by 24.5%. This indicates the influence of additional classes on the formation of combination actions.

The influence of additional classes is realized in an increase in the number of subjects in subgroup B in both groups. Thus, in the experimental group, the number of participants in the noted subgroup increased by 31.6%, in the control group - significantly less - by only 13.2%.

So, the data characterizing changes in the number of children in subgroups A, B and C of both groups allows us to conclude that the main assumption of the study was confirmed. Indeed, second-graders' attendance at additional extracurricular classes in the "Combinatorics" program allowed them to show a higher level of implementation of combination actions at the final diagnostic lesson.

This tendency was clearly manifested in the fact that the number of schoolchildren in the experimental group who used a mixed strategy when solving problems (subgroup B) was significantly (by a statistically significant amount) greater than the number of schoolchildren in this subgroup included in the control group

4. Conclusion.

The experimental work carried out was aimed at clarifying the possibility of creating conditions for improving combination actions among second-grade schoolchildren. Of the 110 people who took part in the study, the control group consisted of 53 people, the experimental group - 57 people, with whom 32 additional classes were conducted in the non-curricular program "Combinatorics". It was shown that the noted activities are an essential condition for improving combination actions.

The data from the experiments of the study characterize the importance of the study for studying the intellectual development of younger schoolchildren when solving problems of developmental and educational psychology.

In the future, it is planned to conduct research aimed at a broader study of the conditions that ensure the improvement of combination activities in primary school.

In particular, it makes serious theoretical and practical sense to organize experimental work with children studying in the third and fourth grades. This should be done in order to characterize the “Combinatorics” program as a condition that ensures the improvement of combination actions in children studying in primary school across a wide age range. range (second – fourth grades).

An important direction for further work is to enrich the problematic material of the Combinatorics program by expanding the types of search tasks offered to children. It is also important to develop and test different approaches to the duration of additional classes in general and their individual episodes in particular. At the same time, the question of the total number of additional classes will be decided depending on the age of the subjects.

It is also of promising research interest to apply the developed and tested “Combinatorics” program in order to create a set of programs based on non-educational material in order to improve critical and creative thinking in younger schoolchildren.

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