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Varlamova A.V. Investigation of natural vibrations of needles embedded in an elastic base

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Abstract. *In this paper, small oscillations of needles on an elastic base are considered, taking into account dissipation in the case of a suddenly applied force, an equation of motion and its solution are proposed, the dynamism coefficient and its maxima are determined.*
Keywords: *oscillations, equation of motion, needle, base, dynamism coefficient.*

1. Introduction.

The aim of this work is to study the vibrations of needles embedded in an elastic base. To determine the dynamic coefficients in the first approximation, we represent the needle in the form of an elastic non-inertial beam, rigidly fixed at one end, carrying a concentrated mass at the point of application of the shock load.

2. Materials and methods

The equation of small oscillations of such a system, taking into account the dissipation in the case of a suddenно applied force, will be as follows [1]:

$$m \cdot \ddot{y} + k \cdot \dot{y} + c \cdot y = P_{ч} \quad (1)$$

or

$$\ddot{y} + 2 \cdot n \cdot \dot{y} + p^2 \cdot y = \frac{P_z}{m} \quad (2)$$

where m is the mass of the needle brought to the point of impact;

k is the damping coefficient associated with the stiffness coefficient by the relation:

$$K = \frac{\psi}{2\pi p} \quad (3)$$

n = $\frac{K}{2m}$; p = $\sqrt{\frac{c}{m}}$ - natural vibration frequency

Absorption coefficient ψ we assume approximately 0.62.

The solution of equation (1) has the form:

$$y = e^{-m}(C_1 \cos pt + C_2 \sin pt) + \frac{P_z}{mp^2} \tag{4}$$

The first two equations describe damped natural oscillations. Differentiate (4):

$$\dot{y} = c_1 e^{-nt}(-p\sin(pt) - n\cos(pt)) + c_2 e^{-nt}(h\cos(pt) - n\sin(pt)) \tag{5}$$

To define arbitrary constants, we have the initial conditions for t=0 y=0, $\dot{y} = 0$, whence

$$c_1 = -\frac{P_r}{mp^2}, c_2 = -\frac{n}{p}\frac{P_r}{mp^2}$$

Then (5) is written as:

$$y = \frac{P_r}{mp^2}\left[1 - e^{-nt}\cos(pt) + \frac{n}{p}\sin(pt)\right] \tag{6}$$

It is easy to see that $\frac{P_r}{mp^2}$ -represents a static movement of mass, hence

$$k_{dr\max i} = 1 + e^{-nt}(\cos(pt) + \frac{n}{p}\sin(pt)) \tag{7}$$

Its successive maxima are determined at the following time points $t = \frac{i\pi}{p}, i = 1,2,3..$:

$$k_{dr\max i} = 1 + e^{-\frac{n}{p}i\pi}$$

Taking into account the introduced notation and dependence (3), the maximum values of the dynamic coefficient are calculated using the formula:

$$k_{dr\max i} = 1 + e^{-\frac{\psi}{4}i} \tag{8}$$

Since the time spent by the needles in the impact zone T_r =(12 ... 60) s is large compared to the period of natural vibrations, the termination of this force does not differ from the removal of static load during static deflection of the needle. Therefore, dynamic phenomena are not of interest at this point.

Let us now proceed to the determination of the dynamic coefficient from the impact on the needley .

The force pulse from their impact on the needle:

$$S = P_y \int_0^{T_0} f(t)dt, \tag{9}$$

where f(t) is the pulse shape,

T_0 -time of impact.

We assume f(t)=1-a rectangular pulse, so

$$p = \frac{S}{T_0} \tag{10}$$

static deflection of the needle under the action of force P_y:

$$y = \frac{P_y}{C} = \frac{P_y}{mp^2} \tag{11}$$

Substituting (10) and (11) in the expression for dynamic deflection, we obtain:

$$y_d = K_d y = \frac{S_0}{mp}, \tag{12}$$

$$\text{where } S_0 = \frac{K_d S}{pT_0} \tag{13}$$

The homogeneous differential equation of vibrations is represented in the form:

$$\ddot{y} + 2n\dot{y} + p^2 y = 0 \tag{14}$$

Its solution, which looks like:

$$y = e^{-nt}(c_1 \cos(pt) + c_2 \sin(pt)), \tag{15}$$

it is a free oscillation with attenuation. Expression (15) under the action on the needle of a certain mass mbmoving with a linear velocity V, that is, the force pulse S=mbv, must satisfy the initial conditions for t=0 y=0, $mb\dot{y} = S_0$. Given them, we find arbitrary constants:

$$c_1 = 0; c_2 = \frac{S_0}{mp}$$

Now (15) takes the form:

$$y = \frac{S_0}{mp} e^{-nt} \sin(pt) = \frac{k_d p_y T_0}{pT_0 mp} e^{-nt} \sin(pt) = k_d y_c e^{-nt} \sin(pt), \tag{16}$$

where does the dynamic coefficient come from?

$$k_{dy} = k_d e^{-nt} \sin(pt)$$

(17)

As before, to find its maximum, we equate the first derivative (17) to zero

$$k_{dy} = k_d e^{-nt}(p\cos(pt) - n\sin(pt)) = 0 \tag{18}$$

This equality is possible when

$$p\cos(pt) - n\sin(pt) = 0 \tag{19}$$

3. Results and Discussion

Let's set aside a vector of value p from the origin pby the angle pt from the abscissa axis in a counterclockwise direction, then a vector of value n it will be positioned at a right angle in the clockwise direction from the first one. The resulting vector is a projection on the x-axis of the vector modulo $=\sqrt{p^2+n^2}$ and composing the angle with the a-b-sciss axis $\left(-pt+arctg\left(\frac{n}{p}\right)\right)$. As a result, the latter equation can be represented in the equivalent form

$$\sqrt{p^2+n^2}\cos\left(-pt+arctg\frac{n}{p}\right)=0 \qquad (20)$$

Its solution $t=\dfrac{-(2j-1)\frac{\Pi}{2}+arctg\frac{n}{p}}{p}$, j=1,2,....

There are two roots in the first period $t_1=\dfrac{-\frac{\Pi}{2}+arctg\frac{n}{p}}{p}$, $t_2=\dfrac{-\frac{3\Pi}{2}+arctg\frac{n}{p}}{p}$

Substitute them in the expression

$$K_{dy}=K_d e^{-nt}\left[-\sin pt\left(p^2-n^2\right)-2np\cos pt\right] \qquad (21)$$

Since p>n, it should be noted that $K_{dy}<0$ when t_1.

Therefore, when $t=\dfrac{-\frac{4j-3}{2}\pi+arctg\frac{n}{p}}{p}$ the dynamic coefficient has maxima .

Transforming (21), we arrive at the formula for the sequence of maxima of the dynamic coefficient when hitting the needle:

$$K_{dy\max i}=\left(1+e^{-\frac{\psi}{4}}\right)e^{\frac{-\psi\left(-\frac{4j-3}{2}\pi+arctg\frac{n}{p}\right)}{4\pi}}\sin\left(\frac{-4j+3}{2}\pi+arctg\frac{n}{p}\right) \quad \text{or}$$

$$K_{dy\max i}=\left(1=e^{-\frac{\psi}{4}}\right)e^{-\frac{\psi\left(-\frac{4j-3}{2}\pi=arctg\frac{\psi}{4\pi}\right)}{4\pi}}\sin\left(-\frac{4j-3}{2}\pi+arctg\frac{\psi}{4\pi}\right)$$

In the case of consecutive (K+1) impacts that occur in the case under consideration , the dynamic coefficient can be can be obtained by superposing functions

(5.39) with different reference points j. If $\frac{T_y P}{2\Pi}$ it is equal to an integer, a pulse resonance occurs:

$$K_{dy\max} = \left(1 + e^{-\frac{\Psi}{4}}\right) \sum_{r=0}^{K} e^{-\frac{\Psi}{4\Pi}\left(-\frac{4j-3}{2}\Pi = \arg tg \frac{\Psi}{4\Pi} - r_0 T_y P\right)} \sin\left(-\frac{4j-3}{2}\mathit{\Pi} + arctg\frac{\psi}{4\mathit{\Pi}} - r_0 T_y P\right)$$

The highest of the highs will be in the lastpeepod. Its value is greaterthe closer the ratio of the shock period to the period of natural vibrations $\frac{T_y P}{2\Pi}$ is to 1.

References

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