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Evaluation of investment activity on the basis of the mathematical model in differential equations

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Аннотация. *В работе представлены положения, касающиеся вопроса построения математической модели динамики экономических показателей на основе дифференциальных уравнений. Приведен вывод основных формул, проведены расчеты для построения функции национального дохода в зависимости от размеров капиталовложений в экономике России за период 2011-2016 гг. Актуальность исследования определяется необходимостью оценки и обоснования проводимой в России экономической политики.*

Ключевые слова: *эконометрика, дифференциальные уравнения, математические модели в экономике, инвестиции, инвестиционные парадоксы.*

Abstract*. The article presents theses concerning the construction of a mathematical model of the economic indicators dynamics based on differential equations. The derivation of the main formulas is given, the calculations for constructing the function of the national income depending on the investments in the Russian economy for the period 2011-2016 are made. The relevance of the study is determined by the need to assess and justify the economic policy pursued in Russia.*

Keywords: *econometrics, differential equations, mathematical models in economics, investments, investment paradoxes.*

1. Introduction

Since 2017, the Government of Russia, the Central Bank, the Ministry of Finance and other government bodies, directly or indirectly influencing economic growth, began actively discussing the need to increase investment in all sectors of the economy. This situation is associated with the stimulating impact of investment on economic growth, and therefore, on general economic welfare. However, we should not forget about the need to consideration the paradoxes of investing. On the one hand, an increase in investment may lead to an investment trap related to a significant increase in the interest rate in the economy. On the other hand, the paradoxes of investing are related to the limits of growth and saturation. In this paper, we consider the rationale for the need for a measured "injection" of investment in the economy, based on mathematical models of the dynamics of economic growth in differential equations. Particularly topical is the question of careful selection of the permissible amount of investment becomes in the conditions of instability of the Russian economy, when the multiplier of autonomous expenditures considering the effect of the accelerator reaches 27 units.[*]

2. Materials and Methods

Let us consider for the beginning a non-lagged[**] model of output dynamics considering investments and disposal of funds. It is described by the following linear first-order differential equation:

$$\begin{cases} y'(t) + k(t)y(t) = u(t), \\ y(0) = y_0, \end{cases} \tag{1}$$

where *y(t)* is the value of the cumulative product produced in the region at the time *t*, *k(t)* is the retirement ratio of the funds (inasmuch as the funds are eliminated, $k = -k\ (t)$, and then the output *y(t)* at time *t* is described by the equation $y\ '(t) = -\ k(t)y(t),\ y(0) = y_0$), *u(t)* is the investment flow at time *t*. Note that this model

[*] Author's calculations.

[**] In this paper, we will consider mathematical models without taking into account time lags, since this simplification allows us to prove that the problems associated with uncontrolled investment growth are related to a greater degree not with their retarded impact, but with more structural properties.

is based on a number of assumptions: unaccounted time lags, the assumption of no restrictions on the modernization of funds, production and sale of products, inflation and the possibility of saving (capitalization of cash generated from the sale of products).

Suppose that the investment takes place according to one of the linear schemes:

$$u_i(t) = a_i t + b_i, \qquad \text{где } \int_0^T u_i(t)dt = U, \qquad i = 1, 2. \qquad (2)$$

where u – the amount of money allocated for investment, a_i – the rate of investment growth (the ratio of retirement of funds), b_i – the change in national income. Let Y_1, Y_2 – the value expression of the output according to the first and second schemes, respectively.

We will find graphs of functions for production according to the above schemes of capital investment. For $k = 0$, the solution of equation (1) can be written in the following form:

$$y_i(t) = \frac{a_i t^2}{2} + b_i t + y_{0,} \qquad i = 1, 2.$$

The volume of output (Y) produced during the period [0, T] is expressed with the help of a definite integral of the form $Y = \int_0^T y(t)dt$.

Then:

$$Y_i = \frac{a_i T^3}{6} + \frac{b_i T^2}{2} + y_0 T, \qquad i = 1, 2. \qquad (3)$$

Let us compare the quantities that characterize output in value terms for the first and second investment schemes. From the formulas (2) we have:

$$\int_0^T (a_i t + b_i)dt = U, \qquad i = 1, 2.$$

Then:

$$\frac{a_i T^2}{2} + b_i T = U, \quad i = 1, 2,$$

Or $a_i = \frac{2(U - b_i T)}{T^2}$.

By substituting these expressions in (2), we obtain:

$$Y_i = U\frac{T}{3} + \frac{b_i T^2}{6} + y_0 T.$$

The values of Y_1 and Y_2 differ only in the second term $\frac{b_i T^2}{6}$. Moreover, if $b_1 > b_2$, then the first of the second summands is greater and $Y_1 > Y_2$ for $k = 0$.

We now consider the case when $k > 0$. In this case, the solution for (1) is the function:

$$y_i(t) = \frac{(a_i kt + b_i k - a_i)}{k^2} + Ce^{(-kt)}, \quad i = 1, 2.$$

A function characterizing the output of Y_i over a period of time [0, T] is a definite integral of the form:

$$\mathrm{Y}_i = \int_0^T y_i(t)dt.$$

Then we have:

$$Y_i = \frac{(a_i k\frac{T^2}{2} + b_i kT - a_i T)}{k^2} + (y_0 k^2 - b_i k + a_i)\frac{(1 - e^{(-kT)})}{k^3}, \qquad i = 1, 2.$$

From formulas (2) we obtain that $a_i = \frac{2(U - b_i T)}{T^2}$, $i = 1, 2$.

Substituting these expressions in (3), we obtain:

$$Y_i = U\frac{(kT - 2)}{Tk^2} + \frac{2b_i}{k^2} + (y_0 k^2 + \frac{2U}{T^2} - b_i(k + \frac{2}{T})(\frac{1 - e^{(-kT)}}{k^3})$$

The numbers differ only in terms containing b_1 and b_2:

$$\frac{2b_1}{k^2} - b_1(k + \frac{2}{T})(\frac{1 - e^{(-kT)}}{k^3})$$

and

$$\frac{2b_2}{k^2} - b_2(k + \frac{2}{T})(\frac{1 - e^{(-kT)}}{k^3}).$$

Accordingly, if $b_1 > b_2$, then the first of these terms is greater than the second and expression

$$\mathrm{K} = 2\mathrm{k} - (\mathrm{k} + \frac{2}{T})(1 - e^{(-kT)})$$

Is more than zero (if k and T are greater than zero). Indeed, K = 0 (the volume of retirement of funds) for $k = 0$, and the derivative

$$\frac{dK}{dk} = 2(T^2 + 1)\frac{1 - e^{(-kT)}}{T^2}$$

equal to zero for $k = 0$ and greater than zero for $k > 0$ and fixed $T > 0$. Hence, for $b_1 > b_2$ we have $Y_1 > Y_2$.

Thus, we get that if there are two linear investment schedules that have the same volume of investment U, the larger volume of output in value terms will be obtained from that investment schedule with a larger initial ordinate [1].

The model considered explains the inertia of a fall in production volumes even after the cessation of a decrease in investment flows.

Let $k = const > 0$ and the continuation of the fall in production volumes lasts up for some time t_0 (years). Investments linearly decrease until this moment, and then increase symmetrically. Then the annual output of Y immediately after the moment t_0 will not grow, because the inertia of the fall in production after the cessation of investment decline is a regularity [1], [5]. From the propositions proved above, we concluded that the larger *Y* will be obtained from a linear investment schedule, the initial ordinate of which is larger. If the volume *Y* falls, when the investment flow increases, this is especially noticeable since $Y(t_0) > Y(t_0-1)$. We also note that the inertial effect of the fall can be dragged on for a long period of up to several decades, unless an appropriate economic policy aimed at changing certain factors of the model is carried out. Thus, to reduce the duration of the period of inertial decline in production volume, an increase in investment or a decrease in the retirement ratio of funds (for example, through the restoration of idle capacities, through modernization of production, etc.) may occur. While the second method is more effective (as can be seen from the derived formulas in which the value of *k* has a more destabilizing effect on production volumes than the amount of investment).

3. Results and Discussion

Table 1

The table shows the deflated* indicators for the period 2011-2016 [7].

Years	Coefficient of retirement of fixed assets	Investments, trillion of rubles	GNI, trillion of rubles
2011	0,8	11,0357	58,4729
2012	0,3	11,5537	60,6107
2013	0,4	12,7783	61,6928
2014	0,2	12,9023	62,1485
2015	1	12,8623	60,3907
2016	0,8	14,0881	60,2549

Based on the data shown above, we constructed the following functions $u_i = u(t)$ using regression analysis:

* Author's calculations.

- $U_{I\,2011-IV\,2013}=10{,}0466+0{,}8713t, \quad t=1, 2, 3;$
- $U_{I\,2014-IV\,2016}=10{,}3197+0{,}5929t, \quad t=4, 5, 6.$

For the purposes of the study, it was decided to construct a piece function in connection with a change in the slope of the trend line of the dynamics of GNI (otherwise the coefficients of the regression model of the dynamics of GNI are statistically insignificant. The division of the periods I quarter 2011 – IV quarter 2013 and I quarter 2014 – IV quarter 2016 is also due to a change in 2014, the economic and political situation in connection with the introduction of sanctions, as a result of which the marginal propensity to import into the Russian economy has decreased, and a significant part of the multiplicative chain has shifted the territory of Russia, causing the accelerator effect to a greater extent in the domestic economy).

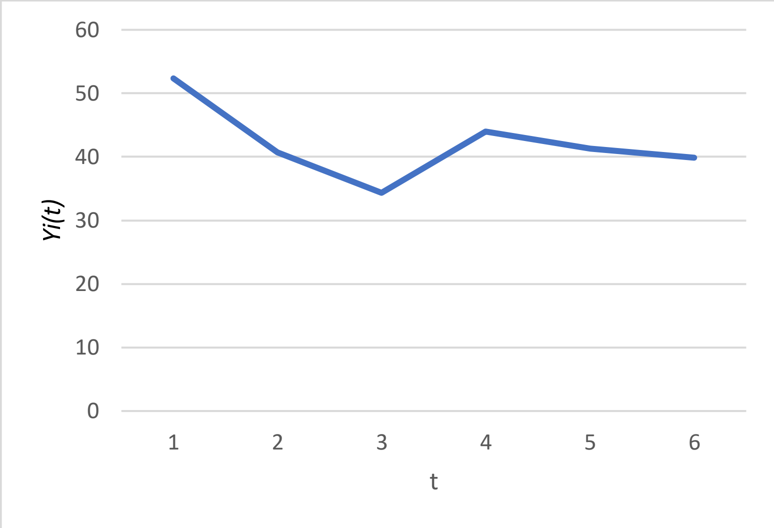
Based on the analysis algorithm and the table data above, we will derive functions for calculating the sizes of GNI in these periods[**] [***]:

- $Y_{I\,2011-IV\,2013} = 1{,}7426t+16{,}608+56{,}0623e^{(-0{,}5t)}, \quad t=1, 2, 3;$
- $Y_{I\,2014-IV\,2016} =1{,}6024t+23{,}5602+61{,}6928e^{(-0{,}37t)}, \quad t=4, 5, 6.$

Substituting the values in the equations, we get that the values of GNI in relation to investments in a given period of time were: 52,3541 in 2011, 40,71737 in 2012, 34,34499 in 2013, 44,01340638 in 2014, 41,27260105 in 2015 and 39,87500003 in 2016 (the dynamics is presented on the graph).

[**] The value of GNI in 2010 ($y_0 = y\,(0)$) was 56.063 trillion rubles.

[***] To simplify the calculations, the average values of the capital outflow coefficients are taken.



Picture1

We note that the data obtained do not coincide with the real ones, since they characterize the model that takes into account only the influence of investment infusions in time and does not take into account a multitude of other factors. However, the model constructed using differential equations makes it possible to clearly demonstrate the effect of the investment paradox, when an increase in capital investment does not always lead to an increase in the national income, and even with sufficiently large capital investments it even contributes to its reduction.

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